

# Screening Length of Rotating Heavy Meson from AdS/CFT

J. Sadeghi · S. Heshmatian

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**Abstract** In this paper we consider a quark-antiquark ( $q\bar{q}$ ) pair which can be interpreted as a meson in  $\mathcal{N} = 4$  SYM thermal plasma. We assume that the string moves at speed  $v$  and rotates around its center of mass simultaneously. By using the AdS/CFT correspondence, we obtain the momentum densities of the rotating string and determine its motion for small angular velocities. Then in general case, we calculate the screening length of  $q\bar{q}$  pair numerically and show that its velocity dependence is in consistent with the well known formula  $L_s T \sim (1 - v^2)^{1/4}$  in the literature.

**Keywords** AdS/CFT correspondence · Super Yang-Mills theory · Black hole · String theory

## 1 Introduction

The AdS/CFT correspondence [27, 42, 56] plays an important role in many complicated problems in QCD at strong coupling. For example, one of these problems is the motion of charged particles through a thermal medium. Already the subject of a quark in thermal plasma at weak coupling has been studied well [3, 6, 16, 17, 32, 44, 46–48, 54]. But QCD at the strong coupling will be a hard problem, however the AdS/CFT correspondence solves the most of these complicated problems. On the other hand in AdS/CFT correspondence there is the relation between type IIB string theory in  $AdS_5 \times S^5$  space and  $\mathcal{N} = 4$  super Yang-Mills (SYM) gauge theory on the 4-dimensional boundary of  $AdS_5$  space, this correspondence will be a candidate for solving these problem. In that case, instead of a quark in gauge theory, we consider dual picture of quark which is an open string in AdS space. By adding temperature to the medium in gauge theory we have a black hole (black brane) in  $AdS_5$  space. There are many interesting works in this field, for example, energy loss of

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quark and drag force on moving quark through  $\mathcal{N} = 4$  Super Yang-Mills thermal plasma [9, 12, 23, 26, 29, 30, 39, 55]. Recently, the same calculations are done for  $\mathcal{N} = 2$  supergravity thermal plasma [52, 53]. This subject is important because the solutions of supergravity theory with  $\mathcal{N} = 4$  and  $\mathcal{N} = 8$  supersymmetry may be reduced to the solutions of  $\mathcal{N} = 2$  supergravity. In Refs. [52, 53] we found that the problem of drag force in  $\mathcal{N} = 2$  supergravity thermal plasma at zero non-extremality parameter is corresponding to  $\mathcal{N} = 4$  SYM plasma for heavy quark.

The most fascinating problem is to consider a  $q\bar{q}$  pair which may be interpreted as a meson. As we know the problem of celebrated Regge behavior of the hadron spectra has been discussed in literature [4, 15, 34, 49]. Also the meson spectrum obtained so far and reasonably describing experiment can be seen in the Ref. [45]. In this Ref. we see that the angular momentum plays an important role to obtain the meson spectrum (in case of correction). So, this give us motivation to consider the rotation of meson.

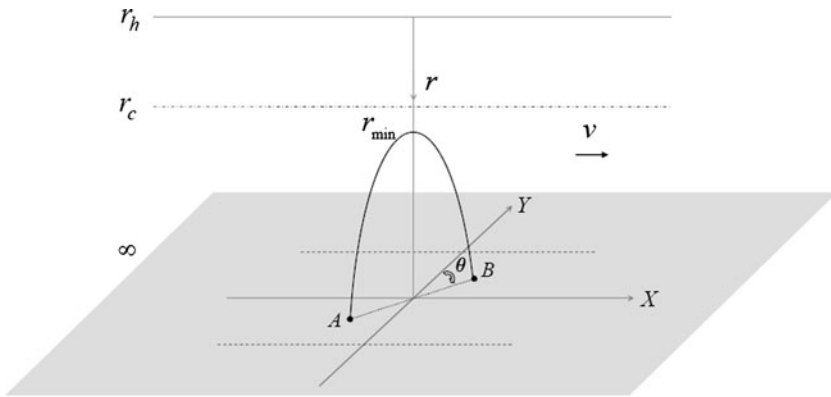
Already, energy of the moving  $q\bar{q}$  pair through  $\mathcal{N} = 4$  SYM plasma is studied in both rest frames of plasma and  $q\bar{q}$  pair, which relate to each other by a Lorentz transformation [14, 41, 51]. Authors in [14] found that the  $q\bar{q}$  pair feels no drag force, so such a system is an ideal system. Actually, the  $q\bar{q}$  pair may have more degrees of freedom such as the rotational motion around the center of mass and oscillation along the connection axis. In the Refs. [19, 31] the description of quark-antiquark system instead single quark well explained. Also the problem of spinning open string (meson) in description of non-critical string/gauge duality [13] considered. In that paper the relationship between the energy and angular momentum of spinning open string for the Regge trajectory of mesons in a QCD-like theory is studied [36, 37].

In this paper, we add a rotational motion to the moving  $q\bar{q}$  pair and obtain the momentum densities  $\Pi_x^1$  and  $\Pi_y^1$  for such system. We assume that  $q\bar{q}$  pair moves at a constant velocity  $v$  along  $X$  direction and rotates simultaneously around its center of mass. In this paper we will obtain effect of rotational motion to the drag force on  $q\bar{q}$  pair. First we find most general solutions for momentum densities and then consider infinitesimal angular velocity to obtain drag force on a heavy  $q\bar{q}$  pair with non-relativistic velocity. We obtain analytical solutions for  $x$  and  $y$  components of string as functions of  $r$  and momentum densities. Our method in this paper differs from Ref. [13]. In Ref. [5] the authors considered a rotating quark and calculated the drag force on it through the plasma.

The calculation of screening length is one of the complicated problems in QCD, but it can be studied in a strongly coupled  $\mathcal{N} = 4$  YM plasma [2, 11, 35, 40]. As we know, the screening length is direction dependent so it is interesting to calculate the screening length in our rotating configuration. In general case (relativistic) the integrals can not be solved analytically, so we use the numerical method suggested in Ref. [35] to determine the screening length at certain velocities. Then we show that the well known formula  $L_s T \sim (1 - v^2)^{1/4}$  is valid in our configuration.

## 2 String Equation of Motion

An open string with two masses at its endpoints can be considered as a model to describe a  $q\bar{q}$  meson. In this picture two point-like masses are connected with a string. This configuration illustrates the strong interaction between two quarks due to the gluon field flux tube and describes the confinement mechanism in QCD. This model can also be used to investigate the orbitally excited states of mesons and baryons. In the classical level, these states can be regarded as the rotation of  $q\bar{q}$  system. It is necessary to consider more complicated motions



**Fig. 1** A rotating U-shape string dual to a  $q\bar{q}$  pair which can be interpreted as a meson.  $A$  and  $B$  represent quark and antiquark with separating length  $L$ . The radial coordinate  $r$  varies from  $r_h$  (black hole horizon radius) to  $r = r_0$  on  $D$ -brane.  $r_c$  is a critical radius, obtained for single quark solution, which the string can not penetrate and  $r_{min} \geq r_c$ .  $r_{min} = r_c$  is satisfied if  $A$  and  $B$  are located at origin ( $L = 0$ ), in that case there is straight string which is the dual picture of a single static quark.  $\theta$  is assumed to be the angle with  $Y$  axis and the string center of mass moves along  $X$  axis at constant velocity  $v$

of string to describe the radial excitations and even other hadron excited states in addition to the rotation. Indeed we have a spinning open string and this picture is interesting because it is to the rotating meson.

In order to represent the quark-antiquark pair, we consider an open string in  $AdS$  space and is stretched from  $D$ -brane to the black hole horizon. The endpoints of string lie on  $D$ -brane and represent quark and antiquark. The string starts from any point on  $D$ -brane in  $X$ - $Y$  plane with radius  $r = r_0$  to  $r = r_{min}$  and then returns to the other point on  $D$ -brane. We suppose the initial condition that at  $t = 0$  the string is upright and its two endpoints move with speed  $v$  along  $X$  axis and rotate around their center of mass in  $X$ - $Y$  plane. The sketch of this string has been shown in Fig. 1.

According to the symmetry, we have  $Y(r_{min}) = 0$  at  $t = 0$  without the rotational motion and two halves of the string are attached together smoothly. The string doesn't lean backward because the drag force along the motion will be zero at  $r = r_{min}$  due to the symmetry. On the other hand, the force along the string should be constant because each segment of string moves at constant velocity, so we conclude that string remains upright [14]. Also there is the Neumann boundary condition  $X'(r_0) = 0$  on  $D$ -brane. At the presence of rotational motion,  $F_X \propto \Pi_X^1$  and  $F_Y \propto \Pi_Y^1$  are constant with respect to  $r$ . That is a consequence of the equation of motion for  $X$  and  $Y$ , therefore the string remains upright.

From Maldacena dictionary we know that adding temperature to the system is equal to the existence of a black hole at the center of  $AdS$  space. For the dual picture of  $\mathcal{N} = 4$  SYM plasma there is the  $AdS_5$  black hole solution which is given by [14],

$$\begin{aligned}
 ds^2 &= \frac{1}{\sqrt{H}}(-hdt^2 + d\vec{x}^2) + \frac{\sqrt{H}}{h}dr^2, \\
 h &= 1 - \frac{r_h^4}{r^4}, \\
 H &= \frac{R^4}{r^4},
 \end{aligned}
 \tag{1}$$

where  $R$  is the curvature radius of AdS space and  $r_h$  is the radius of black hole horizon and  $\vec{x} : (X, Y, Z)$ . The string motion in considered to be in  $X$ - $Y$  plan  $Z = 0$ .

We know that the dynamics of an open string is described by the Nambu-Goto action,

$$S = T_0 \int dt dr \mathcal{L}, \tag{2}$$

where we used static gauge ( $\sigma = r$  and  $\tau = t$ ). Therefore the Lagrangian density of system may be found as,

$$\begin{aligned} \mathcal{L} &= -\sqrt{-g} \\ &= -\left[ 1 + \frac{h}{H}(X'^2 + Y'^2) - \frac{1}{h}(\dot{X}^2 + \dot{Y}^2) - \frac{1}{H}(\dot{X}^2 Y'^2 + \dot{Y}^2 X'^2 - 2\dot{X} X' \dot{Y} Y') \right]^{\frac{1}{2}}, \end{aligned} \tag{3}$$

where prime and dot denote derivative with respect to  $r$  and  $t$  respectively. Also  $g \equiv \det g_{ab}$  where  $g_{ab}$  is the metric on the string world sheet. It is the most general Lagrangian for the string with the endpoints on  $D$ -brane. We see that the Lagrangian density (3) depends on the derivatives of  $X$  and  $Y$ , therefore  $\frac{\partial \mathcal{L}}{\partial X} = \frac{\partial \mathcal{L}}{\partial Y} = 0$ . Then by using the Lagrangian density (3) the string equations of motion are given by the following relations,

$$\frac{\partial}{\partial r} \left[ \frac{1}{H\sqrt{-g}} ((\dot{Y}^2 - h)X' - \dot{X}\dot{Y}Y') \right] + \frac{1}{H} \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( \left( \frac{H}{h} + Y'^2 \right) \dot{X} - \dot{Y}Y'X' \right) \right] = 0, \tag{4}$$

and

$$\frac{\partial}{\partial r} \left[ \frac{1}{H\sqrt{-g}} ((\dot{X}^2 - h)Y' - \dot{X}\dot{Y}X') \right] + \frac{1}{H} \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( \left( \frac{H}{h} + X'^2 \right) \dot{Y} - \dot{X}X'Y' \right) \right] = 0, \tag{5}$$

corresponding to  $X$  and  $Y$  respectively. In order to obtain the total energy and momentum, drag force or energy loss of meson in the thermal plasma, one should first calculate the canonical momentum densities using the following expressions,

$$\begin{pmatrix} \Pi_X^0 & \Pi_X^1 \\ \Pi_Y^0 & \Pi_Y^1 \\ \Pi_r^0 & \Pi_r^1 \\ \Pi_t^0 & \Pi_t^1 \end{pmatrix} = -\frac{T_0}{H\sqrt{-g}} \begin{pmatrix} \dot{Y}Y'X' - \dot{X}Y'^2 - \frac{H}{h}\dot{X} & \dot{X}\dot{Y}Y' - X'\dot{Y}^2 + hX' \\ \dot{X}X'Y' - \dot{Y}X'^2 - \frac{H}{h}\dot{Y} & \dot{X}\dot{Y}X' - Y'\dot{X}^2 + hY' \\ \frac{H}{h}(\dot{X}X' + \dot{Y}Y') & -\frac{H}{h}(\dot{X}^2 + \dot{Y}^2 - h) \\ h(X'^2 + Y'^2 + \frac{H}{h}) & -h(\dot{X}X' + \dot{Y}Y') \end{pmatrix}. \tag{6}$$

If we consider a time dependent rotational motion, in contradistinction to the previous works (without rotation) [14, 29, 30], the time derivative in the second term of (4) and (5) doesn't vanish, so we have a complicated differential equation. But we don't like to solve these equations and it is not the purpose of this paper. Our aim is to determine the momentum densities of the string and obtain the effect of rotational motion on the energy loss and drag force. In the next section we consider a different method with Refs. [5, 13] to study the rotational motion.

### 3 Rotating String

We consider the following ansatz for the string,

$$\begin{aligned} X(r, t) &= vt + x \sin \omega t, \\ Y(r, t) &= y \cos \omega t, \end{aligned} \tag{7}$$

where  $v$  and  $\omega$  are the linear and angular velocities respectively. we choose  $\theta(t) = \omega t$  as the angle with  $Y$  axis as shown in Fig. 1. The functions  $x$  and  $y$  in the right hand of (7) are only depend on  $r$ . Our goal in this paper is to specify the motion of string and calculate the drag force on  $q\bar{q}$  pair. Then we calculate the screening length by using the numerical method. We follow the methods of [14, 29, 30, 35, 52, 53] and put  $\dot{X} = v + \omega x \cos \omega t$ ,  $X' = x' \sin \omega t$ ,  $\dot{Y} = -\omega y \sin \omega t$  and  $Y' = y' \cos \omega t$  in the above equations. By using (4), (5) and (6) one can obtain the following equation,

$$\frac{\partial}{\partial t}(\Pi_X^0 + \Pi_Y^0) + \frac{\partial}{\partial r}(\Pi_X^1 + \Pi_Y^1) = 0, \tag{8}$$

where,

$$\begin{aligned} & \begin{pmatrix} \Pi_X^0 \\ \Pi_Y^0 \\ \Pi_X^1 \\ \Pi_Y^1 \end{pmatrix} \\ &= -\frac{T_0}{H\sqrt{-g}} \\ & \times \begin{pmatrix} -xy'^2\omega \cos^3 \omega t - x'y'y\omega \cos \omega t \sin^2 \omega t - y'^2 \cos^2 \omega t - \frac{H}{h}(v + x\omega \cos \omega t) \\ yx'^2\omega \sin^3 \omega t + (v + x\omega \cos \omega t)x'y' \sin \omega t \cos \omega t + \frac{H}{h}y\omega \sin \omega t \\ x'y^2\omega^2 \sin^3 \omega t - (v + x\omega \cos \omega t)yy'\omega \sin \omega t \cos \omega t + hx' \sin \omega t \\ hy' \cos \omega t - y'(v + x\omega \cos \omega t)^2 \cos \omega t - (v + x\omega \cos \omega t)yx'\omega \sin^2 \omega t \end{pmatrix}, \tag{9} \end{aligned}$$

and,

$$\begin{aligned} -g &= 1 + \frac{h}{H}(x'^2 \sin^2 \omega t + y'^2 \cos^2 \omega t) \\ & - \frac{1}{h}(v^2 + x^2\omega^2 \cos^2 \omega t + 2vx\omega \cos \omega t + y^2\omega^2 \sin^2 \omega t) \\ & - \frac{1}{H}(y'^2 \cos^2 \omega t (v + x\omega \cos \omega t)^2 + x'^2 y^2 \omega^2 \sin^4 \omega t) \\ & - \frac{2}{H}(v + x\omega \cos \omega t)x'y'y\omega \cos \omega t \sin^2 \omega t. \tag{10} \end{aligned}$$

In order to obtain the total energy and momentum of the string one can use the following relations,

$$\begin{aligned} E &= - \int_{r_{min}}^{r_0} dr \Pi_t^0, \\ P_X &= \int_{r_{min}}^{r_0} dr \Pi_X^0, \\ P_Y &= \int_{r_{min}}^{r_0} dr \Pi_Y^0, \tag{11} \end{aligned}$$

where the energy density is given by,

$$\Pi_t^0 = -\frac{hT_0}{H\sqrt{-g}} \left[ x'^2 \sin^2 \omega t + y'^2 \cos^2 \omega t + \frac{H}{h} \right], \tag{12}$$

and the momentum densities  $\Pi_X^0$  and  $\Pi_Y^0$  are given by (9). After determining  $x$  and  $y$  one can obtain the angular momentum of string using the following relation,

$$J = \int_{r_{min}}^{r_0} dr (X\Pi_Y^0 - Y\Pi_X^0). \tag{13}$$

In addition, it is possible to study Regge trajectory by calculating  $\frac{E^2}{J}$  to specify the drag force. In that case we should determine the string motion and it means that we should find explicit expressions for  $x(r)$  and  $y(r)$ . The corresponding equations are complicated, so we consider a heavy meson with small angular velocity to simplify them.

### 4 Momentum Densities

In this section we consider the special case of small angular velocities to find explicit expression for  $x$ ,  $y$  and momentum densities, then we can find drag force on  $q\bar{q}$  pair. Also we calculate the screening length of the rotating  $q\bar{q}$  pair numerically. Here we consider a moving heavy quark-antiquark pair which moves at constant velocity  $v$  along  $X$  axis and rotates simultaneously around its center of mass. The angle with  $Y$  axis  $\theta = \omega t$  and the angular velocity  $\omega \ll 1$ . This assumption corresponds to the motion of a heavy meson with large spin. Indeed in the very large angular momentum limit a semiclassical approximation is valid. In this limit we can write,

$$\sqrt{-g} \approx \left[ 1 - \frac{v^2}{h} + \frac{h}{H}x'^2 \sin^2 \omega t + \frac{h-v^2}{H}y'^2 \cos^2 \omega t \right]^{\frac{1}{2}}, \tag{14}$$

and from (9) one find the following expression for the momentum currents,

$$\begin{pmatrix} \Pi_X^1 \\ \Pi_Y^1 \end{pmatrix} = -\frac{T_0}{H\sqrt{-g}} \begin{pmatrix} hx' \sin \omega t \\ (h-v^2)y' \cos \omega t \end{pmatrix}, \tag{15}$$

where  $\sqrt{-g}$  is given by (14). By using (14) and (15) we can obtain the following equations,

$$\begin{aligned} x' &= \frac{H(h-v^2)}{h \sin \omega t} \Pi_X^1 \sqrt{\frac{1}{(h-v^2)(hT_0^2 - H\Pi_X^{1\,2}) - hH\Pi_Y^{1\,2}}}, \\ y' &= \frac{H\Pi_Y^1}{\cos \omega t} \sqrt{\frac{1}{(h-v^2)(hT_0^2 - H\Pi_X^{1\,2}) - hH\Pi_Y^{1\,2}}}. \end{aligned} \tag{16}$$

It is clear that (16) are only depend on  $r$  and  $t$ . If we don't use the small angular velocity assumption for heavy meson, then the above solutions will depend on  $r$ ,  $t$ ,  $x$  and  $y$ . From [26, 29, 30] we learn that above expressions must be real, this condition for single quark solution [26, 29, 30, 52, 53] yield to the velocity-dependent critical radius,

$$r_c = \frac{r_h}{(1-v^2)^{\frac{1}{4}}}. \tag{17}$$

By using the reality condition in (16) for the quark-antiquark system one can find special radius,  $r_{min}$ , where functions  $x$  and  $y$  are not imaginary, thus string has real energy. by using

square root quantity in (16) one can obtain the following relation for the turning point,

$$r_{min} = \left[ r_h^4 + \frac{1}{2T_0^2(1-v^2)} \left( b + \sqrt{b^2 - 4T_0^2(1-v^2)v^2r_h^4R^4\Pi_X^2} \right) \right]^{\frac{1}{4}}, \tag{18}$$

where we define,

$$b = R^4(1-v^2)\Pi_X^2 + T_0^2v^2r_h^4 + R^4\Pi_Y^2. \tag{19}$$

It is easy to check that  $r_{min} \geq r_c$ . If we consider  $\Pi_X^1 = 0$  ( $L = 0$ ), the special case of  $r_{min} = r_c$  will be satisfied. We note that  $r_{min} = r_c$  corresponds to the single quark solution [14].

Also we have the following useful conditions at  $r = r_{min}$ ,

$$\frac{y'}{x'} = \cot \omega t. \tag{20}$$

It is easily found that at  $\omega \rightarrow 0$  limit, the condition (20) reduces to  $\frac{y'}{x'} \rightarrow \infty$  [14]. Moreover from solution (7) we can see the boundary conditions  $X(r_0, t) = vt \pm \frac{1}{2} \sin \omega t$  and  $Y(r_0, t) = \pm \frac{1}{2} \cos \omega t$  which reduce to previous conditions without rotational motion, namely  $X(r_0, t) = vt$  and  $Y(r_0, t) = \pm \frac{1}{2}$  for  $\omega \rightarrow 0$ . These boundary conditions can also be satisfied with two separated string which move at velocity  $v$  along  $X$  axis and simultaneously swing a circle of radius  $\frac{1}{2}$ .

Specifying these boundary conditions doesn't lead to a unique solution for equation of motion, so we should specify additional conditions for this motion. Here we assume that the string is initially upright, move at velocity  $v$  and rotates around its center of mass, so it doesn't need any external agent to continue its motion.

Therefore by using relation (16), one can obtain the following relation between momentum flows at  $r = r_{min}$ ,

$$\Pi_X^1 = \frac{h(r_{min})\Pi_Y^1}{h(r_{min}) - v^2} \tan^2 \omega t. \tag{21}$$

Therefore one can set momentum currents as following,

$$\begin{aligned} \Pi_X^1 &= \frac{h(r_{min}) \tan^2 \omega t}{\sqrt{H(r_{min})(h(r_{min}) - v^2) + \frac{h(r_{min})\tan^4 \omega t}{H(r_{min})}}}, \\ \Pi_Y^1 &= \sqrt{\frac{h(r_{min}) - v^2}{H(r_{min}) + \frac{h(r_{min})\tan^4 \omega t}{H(r_{min})(h(r_{min}) - v^2)}}}. \end{aligned} \tag{22}$$

As we see the  $\omega = 0$  limit leads us to have  $\Pi_X^1 = 0$  and  $\Pi_Y^1 = \sqrt{\frac{h(r_{min}) - v^2}{H(r_{min})}}$  which is in agreement with the results of Ref. [14]. Now we find that at the presence of rotational motion, there is a non-zero  $\Pi_X^1$  and the value of  $\Pi_X^1$  increases by  $\omega$ . On the other hand by increasing  $\omega$ , the value of  $\Pi_Y^1$  decreases. The maximum value of  $\Pi_X^1$  obtained at  $\omega t \rightarrow \frac{\pi}{2}$  limit, so one can write  $\Pi_X^1 \approx \sqrt{\frac{r_{min}^4 - r_h^4}{L^4}}$  and  $\Pi_Y^1 = 0$ . Using (14) and (16) one can obtain Lagrangian density as following,

$$\mathcal{L} = -\sqrt{\frac{h-v^2}{h}} \left[ 1 + \frac{h}{H} \frac{H(h-v^2)\Pi_X^2 + \Pi_Y^2}{(h-v^2)(hT_0^2 - H\Pi_X^2) - hH\Pi_Y^2} \right]^{\frac{1}{2}}, \tag{23}$$

which reduces to the following expression at  $v^2 \rightarrow 0$  limit (non-relativistic case),

$$\mathcal{L} = - \left[ 1 + \frac{1}{H} \frac{H(r^4 - r_h^4) \Pi_X^2 + r^4 \Pi_Y^2}{(r^4 - r_h^4) T_0^2 - R^4 (\Pi_X^2 + \Pi_Y^2)} \right]^{\frac{1}{2}}. \tag{24}$$

In such limit, we can find analytical expressions for  $x$  and  $y$  from (16) as following,

$$\begin{aligned} x &= \frac{r R^4 \mathcal{D}^{-\frac{1}{2}}}{r_h^2 \sin \omega t} \Pi_X^1 \mathcal{A} \left[ \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, \frac{r^4 T_0^2}{\mathcal{D}} \right], \\ y &= \frac{r R^4 \mathcal{D}^{-\frac{1}{2}}}{r_h^2 \cos \omega t} \Pi_Y^1 \mathcal{A} \left[ \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, \frac{r^4 T_0^2}{\mathcal{D}} \right], \end{aligned} \tag{25}$$

where

$$\mathcal{D} = r_h^4 T_0^2 + R^4 (\Pi_X^2 + \Pi_Y^2), \tag{26}$$

and  $\mathcal{A}$  is Appell function. Thus we success to determine the motion of a non-relativistic meson in the  $\mathcal{N} = 4$  SYM thermal plasma completely. In this case one can use (11) and (13) to show that the ratio of squared energy to angular momentum is proportional to string tension.

### 5 Screening Length

Now we are going to calculate the screening length for a quark-antiquark pair with both linear and rotational motion. For screening length along  $X$  and  $Y$  we use the following relations,

$$\begin{aligned} L_X &= 2 \int_{r_{min}}^{\infty} dr X', \\ L_Y &= 2 \int_{r_{min}}^{\infty} dr Y', \end{aligned} \tag{27}$$

where  $X'$  and  $Y'$  are those in (16). In general case ( $v^2 \neq 0$ ) the integrals can not be solved analytically and we should use the numerical method to solve them.

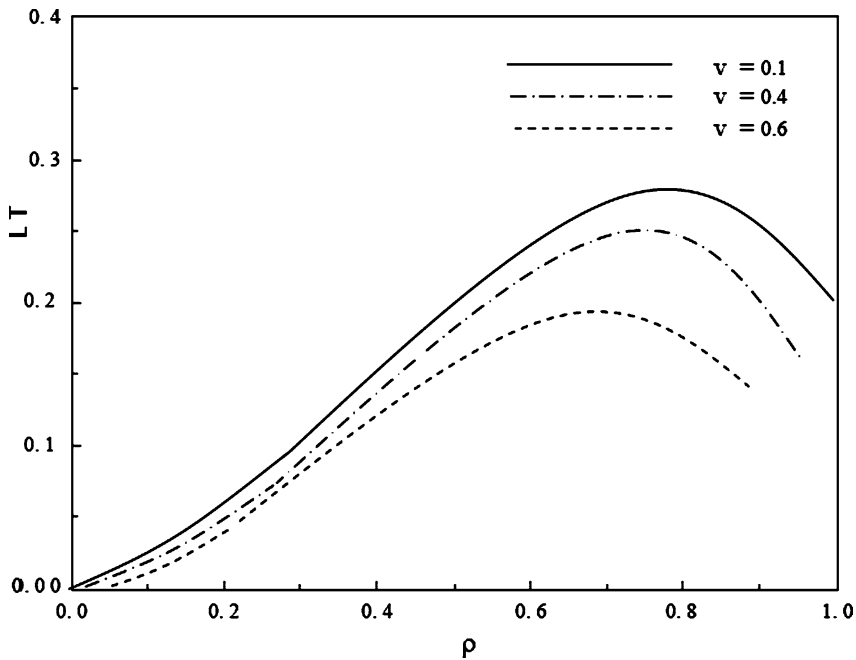
Here we follow the notation of Ref. [35] and change the variables as,

$$\alpha = \frac{\Pi_X^1 L^2}{r_{min}^2}, \quad \beta = \frac{\Pi_Y^1 L^2}{r_{min}^2}, \quad z = \frac{r}{r_{min}}, \quad \rho = \frac{r_h}{r_{min}}. \tag{28}$$

Applying these variables to (16) and use the hawking temperature relation, we can write (27) in terms of new variables and as following,

$$\begin{aligned} L_X &= 2 \int_1^{\infty} dz \frac{\alpha \rho ((1 - v^2) z^4 - \rho^4)}{\pi T r_{min} \sin(\theta) z^4 (z^4 - \rho^4)} \mathcal{F}(z), \\ L_Y &= 2 \int_1^{\infty} dz \frac{\beta \rho ((1 - v^2) z^4 - \rho^4)}{\pi T r_{min} \cos(\theta) z^4} \mathcal{F}(z), \end{aligned} \tag{29}$$





**Fig. 2**  $LT$  vs.  $\rho$  for some values of  $v$ . The maximum is considered as screening length  $L_s$ . According to this diagram one can find that as  $v$  increases, the value of  $L_s$  gets smaller

where  $\mathcal{F}(z)$  has the following form,

$$\mathcal{F}(z) = z^4 [T_0^2(z^4 - \rho^4)((1 - v^2)z^4 - \rho^4) + \rho^4(\alpha^2 + \beta^2) - z^4(\alpha^2 + \beta^2 - \alpha^2 v^2)]^{-\frac{1}{2}}. \tag{30}$$

Now we are able to solve the two equations obtained in previous section simultaneously by the numerical method suggested in [35]. First we choose  $\theta = \pi/6$ , then for each value of  $v$  in the range (0,1) we vary  $\rho$  between 0 and 1. We set  $\alpha$  and  $\beta$  in a way that the two integrals in (28) have the same values. Finally, by inserting these values of  $\rho$ ,  $\alpha$  and  $\beta$  in (28) we obtain the value of  $LT$  and then plot  $LT$  as a function of  $\rho$ . As we expect, for a few values of  $v$  this diagram has a peak which is interpreted as the screening length  $L_s$  of the rotating  $q\bar{q}$  pair, Fig. 1.

Here we demonstrate the velocity dependence of screening length. From Fig. 2 it can be easily found that  $L_s T \sim (1 - v^2)^{1/4}$  is valid. According to this graph as  $v$  increases,  $L_s$  decreases and this result is in consistent with the corresponding formula. Here we find a similar behavior for  $LT$  vs.  $\rho$  as in Ref. [35]. Our result is also in a good agreement with the well known formula for screening length and we find that  $L_s T (1 - v^2)^{-1/4} \simeq 1.1$  for the three various values of  $v$  in Fig. 2.

### 6 Conclusion

This article is an extension of moving  $q\bar{q}$  pair through  $\mathcal{N} = 4$  SYM thermal plasma which has done for the first time (we follow different way with Refs. [5, 13] and also the calculation of the screening length for a rotating  $q\bar{q}$  pair. Already, the same problem for  $v = 0$  and  $v \neq 0$

is studied without the rotation by using AdS/CFT correspondence [14]. Now we consider the rotation of  $q\bar{q}$  pair around their center of mass and obtain the momentum flow along the string which are proportional to the drag force. In the case of  $\theta = \omega t$  at  $\omega \ll 1$  limit for non-relativistic velocities we could specify the total motion of the system. Without the rotation one can obtain the momentum flows  $\Pi$  proportional to the constant  $C$  [14, 29, 30, 52, 53]. But in the case of rotational motion, we found the momentum flows proportional to  $C \tan \omega t$  for small angular velocities. We have shown that for the spinning string with very large angular momentum, the value of momentum current along the string increases in  $X$  direction and decreases in  $Y$  direction, so the maximum value of  $\Pi_X^1$  can be achieved for  $\omega t \rightarrow \frac{\pi}{2}$ , where  $\Pi_Y^1 = 0$ .

Then we calculated the screening length for the considered configuration numerically and checked that the  $L_s T \sim (1 - v^2)^{1/4}$  is valid for three various velocities. From Fig. 2 one can easily find that  $L_s T (1 - v^2)^{-1/4} \simeq 1.1$  for the three various values of  $v$ . The value of  $L_s T$  gets smaller as  $v$  increases and it is the result we expected before.

Here, there are some interesting problem for future works. For example one can obtain shear viscosity [43] or jet quenching parameter [1, 10, 18, 21, 38] for rotating  $q\bar{q}$  pair. Also it is interesting to consider the effect of higher derivative terms as in the previous cases [7, 8, 20, 22, 24, 25, 28, 33, 50]. As a recent work [35] one may consider more quarks, such as four quarks in the baryon through  $\mathcal{N} = 4$  SYM thermal plasma. It may be interesting to consider fluctuations of the quark-antiquark pair and obtain the exact solution of such a system.

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